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# Renormalization by gravity and the Kerr spinning particle

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## Abstract

On the basis of the Kerr spinning particle, we show that the mass renormalization is perfectly performed by gravity for an arbitrary distribution of source matter. A smooth regularization of the Kerr–Newman solution is considered, leading to a source in the form of a rotating bag filled by a false vacuum. It is shown that gravity controls the phase transition to an AdS or dS false vacuum state inside the bag, providing the mass balance.

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## 1. Introduction

Many of complicated quantum procedures admit an interpretation in terms of some classical analogues. The mass renormalization in QED represents a peculiar case. It is universally recognized due to an incredible exactness of its predictions, and its origin lies in the classical theory of a pointlike electron; nevertheless, there are serious problems with physical interpretation and mathematical correctness of this procedure.

In this paper, we consider a semiclassical model of a spinning particle based on the Kerr–Newman solution of the Einstein–Maxwell theory. This solution has a double gyromagnetic ratio, as that of the Dirac electron and may be considered as a model of electron in general relativity [1–4].

In this paper we would like to show that the mass renormalization and regularization of the singularities in the Kerr–Newman source are perfectly realized by gravitational fields in a very natural manner. It allows one to conjecture that the methodological problems of QED may be related to the ignorance of gravity. QED ignores the gravitational field arguing that its local action is negligible. It is true, but only partially. The Kerr solution gives a counter-example to this assertion, showing that the local action of the gravitational field may extend on the Compton distances due to the stringy structure of the source. However, the main effect of gravity is apparently related to a non-local action. We would like to show here that in the semiclassical model of the Kerr spinning particle, gravity provides the mass renormalization.

## 2. Renormalization by gravity

The mass of an isolated source is determined only by an asymptotic gravitational field, and, therefore, it depends only on the mass parameter  $m$  which survives in the asymptotic expansion for the metric. On the other hand, the total mass can be calculated as a volume integral, which takes into account densities of the electromagnetic energy  $\rho_{em}$ , material (mechanical mass) sources  $\rho_m$  and energy of the gravitational field  $\rho_g$ . The last term is not taken into account in QED, but it provides perfect renormalization. For a spherically symmetric system, the expression may be reduced to an integral over radial distance  $r$ ,

$$m = 4\pi \int_0^\infty \rho_{em} dr + 4\pi \int_0^\infty \rho_m dr + 4\pi \int_0^\infty \rho_g dr. \quad (1)$$

It looks like the expressions in a flat spacetime. However, in the Kerr–Schild background, it is a consequence of the exact Tolman relations taking into account energy of matter, energy of gravitational field (including the contribution from pressure) and rotation [6]. In the well-known classical model of an electron as a charged sphere with electromagnetic radius  $r_e = \frac{e^2}{2m}$ , integration in (1) is performed in the diapason  $[r_0, \infty]$ , where  $r_0 = r_e$ . The total mass is determined by electromagnetic contribution only, and contribution from gravity turns out to be null. However, if  $r_0 < r_e$ , the electromagnetic contribution exceeds the total mass and this extension is to be compensated by the negative gravitational contribution. Indeed, the results will not depend on the cut parameter  $r_0$  and, moreover, on radial distribution of matter at all. Some of the terms may be divergent, but the total result will not be changed, since divergences will always be compensated by a contribution from a gravitational term.

It shows that, due to the strong non-local action, gravity may be essential for elementary particles, on distances which are very far from the usually considered Planck scale.

## 3. Structure of the Kerr geometry

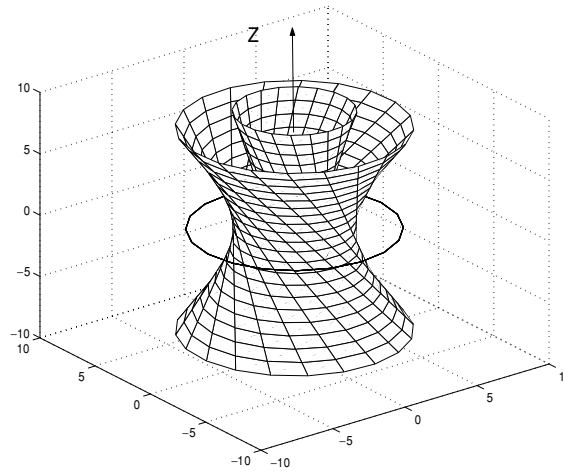
The Kerr–Newman solution breaks the prevailing point of view that the local action of the gravitational field of a particle extends to its Schwarzschild radius. The Schwarzschild singular points turn in the Kerr rotating geometry into a singular ring which extends on the Compton sizes, since its radius  $a = J/m$ , for  $J \sim \hbar$ , is the Compton one, which exceeds the Schwarzschild one for an electron at  $\sim 10^{22}$ . The angular momentum  $J = \hbar/2$  for parameters of electron is so high that the black-hole horizons disappear, and the source of the Kerr spinning particle represents a naked singular ring which may have some stringy excitations, generating the spin and mass of the extended particle-like object—‘microgeon’ [3]. Therefore, the Kerr source represents a closed singular string of the Compton size, and cannot be localized in the region which is smaller than the Compton size<sup>1</sup>. It was shown that this source is indeed a string [3, 7, 8] resembling a heterotic string of superstring theory<sup>2</sup>. Note that this singularity is a branch line of the Kerr space which turns out to be two-sheeted, and the disc spanned on this ring plays the role of gates to anti-world (‘negative’ sheet), where the signs of charges and masses, and the directions of the fields are changed<sup>3</sup>. So, the Kerr string is an ‘Alice’ one, and all the fields have to fill these ‘gates to anti-world’ which have the giant Compton sizes ( $\sim 10^{-11}$  cm). Note that in QED it is the region of virtual photons.

One more remarkable structure of the Kerr geometry is PNC (principal null congruence). It is a vortex of the lightlike rays (twistors) which fall on the ‘negative sheet’ on the Kerr disc,

<sup>1</sup> In this respect, the Kerr source is similar to the Dirac wavefunction.

<sup>2</sup> Adding a stringy tension  $T$  to the Kerr source,  $E \equiv m = Ta$ , and combining this relation with  $J = ma$ , one obtains the Regge dependence  $J = \frac{1}{T}m^2$ .

<sup>3</sup> It was discussed many times, see for example [8, 9].



**Figure 1.** The Kerr singular ring and 3D section of the Kerr principal null congruence (PNC). The singular ring is a branch line of space, and PNC propagates from the ‘negative’ sheet of the Kerr space to the ‘positive’ one, covering the spacetime twice.

penetrate it and turn into outgoing ‘out’-fields on the ‘positive sheet’ of space (see figure 1). PNC is a very important object since the tangent to congruence vector  $k^\mu$  determines the Kerr–Schild ansatz for metric

$$g^{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^\nu \quad (2)$$

(where  $\eta^{\mu\nu}$  is the auxiliary Minkowski metric) and the form of vector potential

$$A_\mu = \mathcal{A}(x)k_\mu, \quad (3)$$

for electrically charged solution, i.e. it determines polarization of the gravitational and electromagnetic fields around the Kerr source and the directions of radiation for the non-stationary excited solutions [7, 9].

The Kerr congruence is determined by the Kerr theorem [9–11] in terms of twistors. The Kerr singular ring is a focal line of the Kerr PNC.

The Kerr–Schild form of metric allows one to consider a broad class of regularized solutions which remove the Kerr singular ring, covering it by a matter source. There is a long-term story of the attempts to find some interior regular solution for the Kerr or Kerr–Newman solutions [1, 4–6]<sup>4</sup>. Usually, the regularized solutions have to retain the Kerr–Schild form of metric and the form of Kerr principal null congruence  $k_\mu(x)$ , as well as its property to be geodesic and shear-free. The space part  $\vec{n}$  of the Kerr congruence  $k_\mu = (1, \vec{n})$  has the form of a spinning hedgehog. Indeed, by setting the parameter of rotation  $a$  equal to zero, the Kerr singular ring shrinks to a singular point, and  $\vec{n}$  takes the usual hedgehog form which is used as an ansatz for the solitonic models of elementary particles and quarks. It suggests that the Kerr spinning particle may have relation not only to electron but also to the other elementary particles. Indeed, the Kerr–Schild class of metric has a remarkable property, allowing us to consider a broad class of the charged and uncharged, the spinning and spinless solutions from a unified point of view.

<sup>4</sup> Extra references may be found in [5, 6].

#### 4. Regularization of the Kerr singularity

Our treatment will be based on the approach given in [5, 6], where the *smooth* regularized sources were obtained for the rotating and non-rotating solutions of the Kerr–Schild class. These smooth and regular solutions have the scalar function  $H$  of the general form

$$H = f(r)/(r^2 + a^2 \cos^2 \theta). \quad (4)$$

For the Kerr–Newman solution function,  $f(r)$  has the form

$$f(r) \equiv f_{\text{KN}} = mr - e^2/2. \quad (5)$$

Regularized solutions have three regions:

- (i) the Kerr–Newman exterior,  $r > r_0$ , where  $f(r) = f_{\text{KN}}$ ;
- (ii) interior  $r < r_0 - \delta$ , where  $f(r) = f_{\text{int}}$  and function  $f_{\text{int}} = \alpha r^n$ , and  $n \geq 4$  to suppress the singularity at  $r = 0$ , and provide the smoothness of the metric up to the second derivatives;
- (iii) a narrow intermediate region  $r \in [r_0 - \delta, r_0]$  which allows one to get a smooth solution interpolating between regions (i) and (ii).

It is advisable to first consider the non-rotating cases, since the rotation can later be taken into account by an easy trick. In this case, taking  $n = 4$  and the parameter  $\alpha = 8\pi\Lambda/6$ , one obtains for the source (interior) a spacetime of constant curvature  $R = -24\alpha$  which is generated by a source with energy density

$$\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2, \quad (6)$$

and tangential and radial pressures

$$p_{\text{rad}} = -\rho, \quad p_{\text{tan}} = \rho - \frac{1}{8\pi}f''/\Sigma, \quad (7)$$

where  $\Sigma = r^2$ . It yields for the interior the stress–energy tensor  $T_{\mu\nu} = \frac{3\alpha}{4\pi} \text{diag}(1, -1, -1, -1)$ , or

$$\rho = -p_{\text{rad}} = -p_{\text{tan}} = \frac{3\alpha}{4\pi}, \quad (8)$$

which generates a de Sitter interior for  $\alpha > 0$  and an anti-de Sitter interior for  $\alpha < 0$ . If  $\alpha = 0$ , we have a flat interior which corresponds to some previous classical models of an electron, in particular, to the Dirac model of a charged sphere and to the Lopez model in the form of a rotating elliptic shell [4].

The resulting sources may be considered as the bags filled by a special matter with positive ( $\alpha > 0$ ) or negative ( $\alpha < 0$ ) energy density<sup>5</sup>.

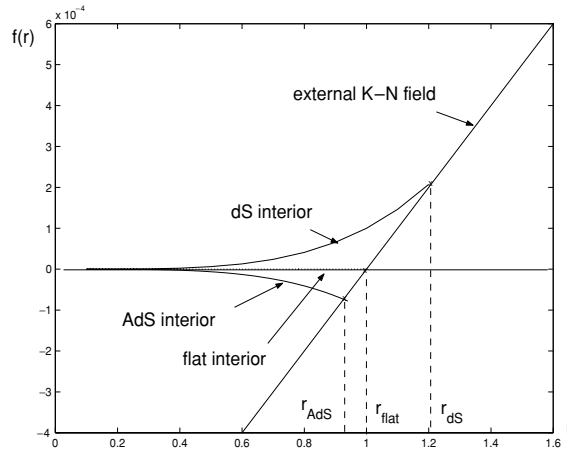
The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from ‘true’ to ‘false’ vacuum in a supersymmetric  $U(1) \times \tilde{U}(1)$  Higgs model [5].

Assuming that the transition region (iii) is very thin, one can consider the following graphical representation which turns out to be very useful, see figure 2.

The point of phase transition  $r_0$  is determined by the equation  $f_{\text{int}}(r_0) = f_{\text{KN}}(r_0)$ , which yields  $\alpha r_0^4 = mr_0 - e^2/2$ . From (8), we have  $\rho = \frac{3\alpha}{4\pi}$  and obtain the equation

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \quad (9)$$

<sup>5</sup> It resembles the discussed at present structure of dark energy and dark matter in universe. The case  $\alpha > 0$  is reminiscent of the old Markov suggestions to consider particle as a semiclosed universe.



**Figure 2.** Regularization of the Kerr spinning particle by matching the external field with dS, flat or AdS interior.

In the first term on the right-hand side, one can easily recognize the electromagnetic mass of a charged sphere with radius  $r_0$ ,  $M_{\text{em}}(r_0) = \frac{e^2}{2r_0}$ , while the second term is the mass of this sphere filled by a material with a homogeneous density  $\rho$ ,  $M_m = \frac{4}{3}\pi r_0^3 \rho$ . Thus, the point of intersection  $r_0$  acquires a deep physical sense, providing an energy balance by the mass formation. In particular, for the classical Dirac model of a charged sphere with radius  $r_0 = r_e = \frac{e^2}{2m}$ , the balance equation yields the flat internal space with  $\rho = 0$ . If  $r_0 = r_{\text{dS}} > r_e$ , the interior is de Sitter space, and a material mass of positive energy  $M_m > 0$  gives a contribution to the total mass  $m$ . If  $r_0 = r_{\text{AdS}} < r_e$ , this contribution has to be negative  $M_m < 0$ , which is accompanied by the formation of an AdS internal space.

### 5. Transfer to a rotating case

All the above treatments are valid for the rotating cases, and for the passage to a rotating case, one only has to set

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad (10)$$

and consider  $r$  and  $\theta$  as the oblate spheroidal coordinates [6]. It looks wonderful, however it is a direct consequence of the structure of function  $H$ , in which the nominator is independent of the rotation parameter  $a$ .

The Kerr–Newman spinning particle with a spin  $J = \frac{1}{2}\hbar$  acquires the form of a relativistically rotating disc which foliates on the rigidly rotating ellipsoidal shells, and the board of the disc has  $v \sim c$  [6]. The corresponding stress–energy tensor (8) describes in this case the matter of source co-rotating with this disc coordinate system. The disc has the form of a highly oblate ellipsoid with thickness  $r_0$  and radius  $a = \frac{1}{2}\hbar/m$ , which is of order of the Compton length. The interior of the disc represents a ‘false’ vacuum having superconducting properties [4, 5], so the charges are concentrated on the surface of this disc, at  $r = r_0$ . Inside the disc, the local gravitational field is negligible.

### 6. Non-stationarity and zero-point radiation

Classical models of a spinning particle encounter an unavoidable conflict with quantum theory. The Kerr singular string acquires electromagnetic wave excitations [3, 7, 8]. In classical theory,

these excitations lead to a radiation which breaks axial symmetry of the Kerr–Newman solution and leads to non-stationarity. As a result, only an average metric takes the Kerr–Newman form. In the Kerr–Schild formalism [2], electromagnetic excitations are related to a field  $\gamma(x)$  which induces electromagnetic radiation along the Kerr congruence  $k_\mu$  and non-stationarity of the solutions. This radiation also leads to infrared divergence of the mass, and there are arguments that this radiation has to be renormalized [3, 7, 8], setting the field  $\gamma = 0$ . In quantum theory, oscillations are stationary and the absence of radiation caused by oscillations is postulated, although the radiation is present in QED too, being related to radiative corrections: the field of virtual photons, vacuum zero-point field and vacuum polarization.

In a semiclassical approach, one can use the receipt of the quantum field theory in curved spaces [12], which takes into account the quantum effects concentrated in the divergent vacuum zero-point field. By the transfer to the classical Einstein–Maxwell theory, these quantum vacuum fields have to be subtracted from the classical stress–energy tensor by a regularization [12],

$$T_{\mu\nu}^{(\text{reg})} = T_{\mu\nu} - \langle 0|T_{\mu\nu}|0\rangle, \quad (11)$$

which has to satisfy the condition

$$T^{(\text{reg})\mu\nu}_{,\mu} = 0. \quad (12)$$

It was conjectured in [7–9] that regularization of the Kerr–Newman stress–energy tensor has to be related to a subtraction of electromagnetic radiation caused by the field  $\gamma$  which propagates along the Kerr congruence  $k_\mu$ , and involves non-stationarity by a loss of mass. Twofoldedness of the Kerr geometry confirms this point of view, since *the outgoing radiation on the ‘positive’ out-sheet of the metric is compensated by an ingoing radiation on the ‘negative’ in-sheet*. It shows that the field  $\gamma$  has to be identified with the vacuum zero-point field and may be subtracted from the stress–energy tensor by means of regularization, which has to satisfy condition (12). Such regularization may be performed [7, 8] and leads to some modified Kerr–Schild equations [7]. It shows that electromagnetic excitations on the Kerr background are similar to the Casimir effect and may be interpreted as a resonance of the zero-point fluctuations on the (superconducting) source of the Kerr spinning particle [7, 8].

Although the exact non-trivial solutions of the regularized system have not been obtained so far, there were obtained corresponding exact solutions of the Maxwell equations which show that any ‘aligned’ excitation of the Kerr geometry leads to the appearance of some extra ‘axial’ singular lines (strings) which are semiinfinite and modulated by the de Broglie periodicity [7, 8]. The recently obtained multiparticle Kerr–Schild solutions [11] support this point of view, leading to the conclusion that the radiating twistorial structure of the Kerr PNC belongs to the vacuum zero-point field, pointing out on the twistorial texture of vacuum [7, 13].

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